**Recitation 4**

**Problem 1.**

There is a pond. Inside the pond there are n pebbles, arranged in a cycle. A frog is sitting on one of the pebbles. Whenever he jumps, he lands exactly k pebbles away in the clockwise direction, where 0 < k < n. The frog’s meal, a delicious worm, lies on the pebble right next to his, in the clockwise direction.

1. Describe a situation where the frog can’t reach the worm.   
     
   It works when , so a situation in which it wouldn’t work would be something like and . The frog would go hit the same pebbles every round, and they would always be even.
2. In a situation where the frog can actually reach the worm, explain how to use the Pulverizer to find how many jumps the frog will need.  
     
   If then the .   
   The frog will reach the worm after any number of steps, plus one. And it will require a number of -long jumps to do this. So, . This can be rewritten as a linear combination, . Since the linear combination equals positive one, . The Pulverizer could then be used to find the terms and for a given pair of . would represent the number of required jumps.
3. Compute the number of jumps if and . Anything strange? Can you fix it?  
     
   Scribbling out the Pulverizer on another sheet gives for . The strange thing is that this is a negative value. We can bring it back into the positive range while maintaining its congruency to the remainder of by adding , giving 31.

**Problem 2**.

Give an inductive proof that the Fibonacci numbers and are relatively prime for all .

*Proof*. By induction. Let be that the Fibonacci numbers and are relatively prime for all

*Base case.* and are 0 and 1, respectively, and is 1, making them relatively prime by the definition in the book.

*Inductive step.* We assume , so by and the definition of relative primes, . From this we will show that , and therefore First we observe the definition of :

We seek to find the value of which we can now express as

By we know so the only divisors of are 1 and itself. And when after the base case. Therefore is not divisible by which means

showing Thus by the induction is true for all .